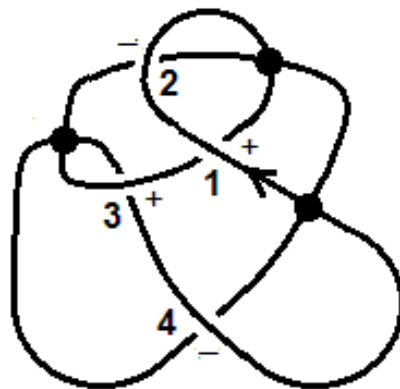


# Classification of Virtual Knots

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# 1. Introduction

Virtual knots [1, 2] are a generalisation of classical knots where one or more of the crossings are replaced by virtual crossings. One way of describing a knot is by using the Gauss code.

## 2. Gauss code

In fig. 1 a projection of the classical knot  $6_3$  is shown. If the string is followed around the knot the crossings are numbered in the order they are encountered. Starting at the arrow the sequence **123453612545** is obtained. If furthermore we distinguish over and under crossings the sequence **O1U2O3U4O5U3O6U1O2U5O4U5**. Finally we distinguish between positive and negative crossings (figure 2) and arrive at the full Gauss code

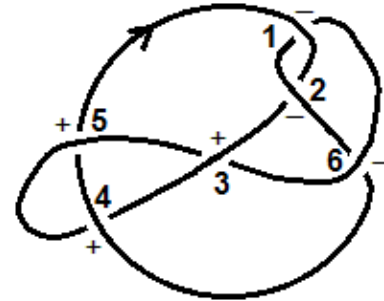


Figure 1: The knot  $6_3$ .

**O1-U2-O3+U4+O5+U3+O6-U1-O2-U6+O4+U5+.**

However we could have started our journey around the knot starting somewhere else and perhaps reversed the direction. In this case the Gauss code would look quite different. From all possible codes one is normally selected. Here we choose the sequence

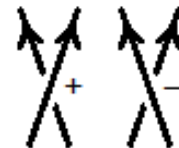


Figure 2: Positive and negative crossings.

**O1+U2+O3+U1+O4-U5-O2+U3+O6-U4-O5-U6-,**

which is the first code if all possible codes are ordered as the smallest number, O before U, and + before -. This code is achieved in figure 1 if the knot is mirrored and the direction is kept but the starting point is between the crossings 3 and 6. The numbering has of course to be changed accordingly.

All classical knots can be described by Gauss codes, but all Gauss codes do not correspond to classical knots.

## 3. Virtual crossings and virtual knots

Let us consider the Gauss code **O1+U2+U1+O2+**. If we try to make a knot out of this we run into problems as shown in fig. 3. There is no way to unite the two ends without crossing the string again. We have to make a virtual crossing, thereby constructing the first virtual knot. The virtual crossing is marked as a black dot and can be considered as not being there.



Figure 3: A virtual knot

## 4. Reidemeister moves and Omega moves

For classical knots we have the three Reidemeister moves [3] (fig. 4) which also applies to virtual knots.

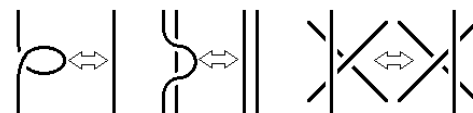


Figure 4: Reidemeister moves.

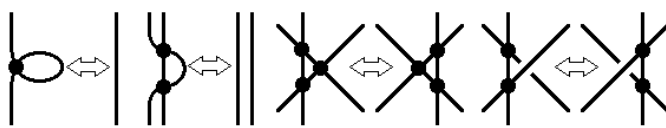


Figure 5: Omega moves.

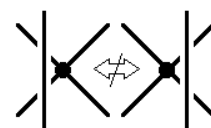


Figure 6: Forbidden move.

There are additional moves, the Omega moves [4], that can be applied to virtual knots, shown in fig. 5. There is also a forbidden move shown in fig. 6. This last move would have changed the order of the real crossings and may thus have changed the knot.

## 6. Parity signature

Let us again consider the Gauss code

$$\mathbf{O1+U2+O3+U1+O4-U5-O2+U3+O6-U4-O5-U6-}.$$

Now consider only the first time a certain crossing is approached. We will then have the sequence  $\mathbf{O1+U2+O3+O4-U5-O6-}$ . Even parity is given to crossings of the types  $\mathbf{O+}$  and  $\mathbf{U-}$ . Odd parity is given to crossings of the types  $\mathbf{O-}$  and  $\mathbf{U+}$ . With  $\mathbf{+}$  for even parity and  $\mathbf{-}$  for odd parity, we obtain the parity signature for this Gauss code as  $\mathbf{+-+--+}$ . Note that the choice of Gauss code (as discussed in section 2) may change the parity signature.

## 7. Classification of virtual knots

Each knot can take eight different forms (including the orientation of the knot). Some of them may however be identical. To invert a knot means to reverse the orientation of a knot and the original Gauss code reads backwards. To take the vertical mirror image of a knot (changing all over crossings to under crossings and vice versa) changes all  $\mathbf{O}$  's to  $\mathbf{U}$  's and vice versa and all  $\mathbf{+}$  's to  $\mathbf{-}$  's and vice versa in the original Gauss code. Finally a horizontal mirror image (look at the knot through a mirror) of a knot just changes the all  $\mathbf{+}$  's to  $\mathbf{-}$  's and vice versa in the original Gauss code. Combining these procedures can produce eight distinct knots. In the following they are all considered to be the same knot. The numbering of the knots are from Jeremy Green's Virtual Knot Table [5]. Note that only signatures starting with  $\mathbf{+}$  are shown since those starting with  $\mathbf{-}$  are obtained by changing all signs. Note that even if the number sequence is given, owing to symmetries, the signature is not always unique. In the following compilation all knots are shown with the minimal number of virtual crossings.

2 real crossings: 1 class, 1 knot

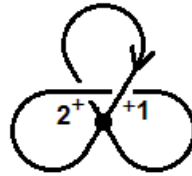
Sequence **1212**

Signatures ++ and +-



**2Aa1**, Green's 2.1

$O_1+O_2+U_1+U_2+$



3 real crossings: 3 classes, 7 knots

Class A: 2 types, 4 knots

Sequence **121323**

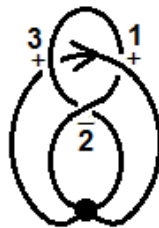


Type a: 1 knot

Signatures +++ and +-+

**3Aa1**, Green's 3.2

$O_1+O_2-U_1+O_3+U_2-U_3+$

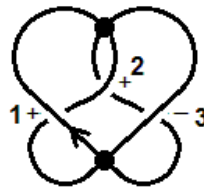


Type b: 3 knots

Signatures ++- and +--

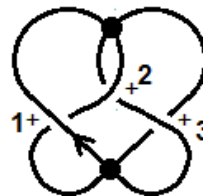
**3Ab1**, Green's 3.1

$O_1+O_2+U_1+O_3-U_2+U_3-$



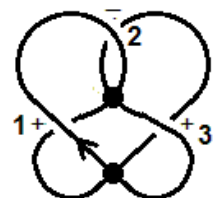
**3Ab2**, Green's 3.3

$O_1+O_2+U_1+U_3+U_2+O_3+$



**3Ab3**, Green's 3.4

$O_1+O_2-U_1+U_3+U_2-O_3+$



Class B: 2 types, 3 knots

Sequence **123123**

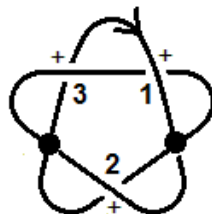


Type a: 2 knots

Signatures +++ ++- and +--

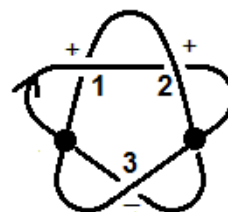
**3Ba1**, Green's 3.7

$O_1+O_2+O_3+U_1+U_2+U_3+$



**3Ba2**, Green's 3.5

$O_1+U_2+O_3-U_1+O_2+U_3-$

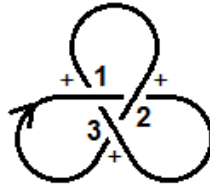


Type b: 1 knot

Signature  $+-+$

**3Bb1**, Green's 3.6

$O_1+U_2+O_3+U_1+O_2+U_3+$



4 real crossings: 7 classes, 109 knots (and 10 duplicates)

Class A: 5 types, 18 knots

Sequence **12123434**



Type a: 5 knots

Signature  $++++$

**4Aa1**, Green's 4.1

$O_1+O_2+U_1+U_2+O_3+O_4+U_3+U_4+$

**4Aa2**, Green's 4.4

$O_1+O_2+U_1+U_2+O_3+U_4-U_3+O_4-$

**4Aa3**, Green's 4.8

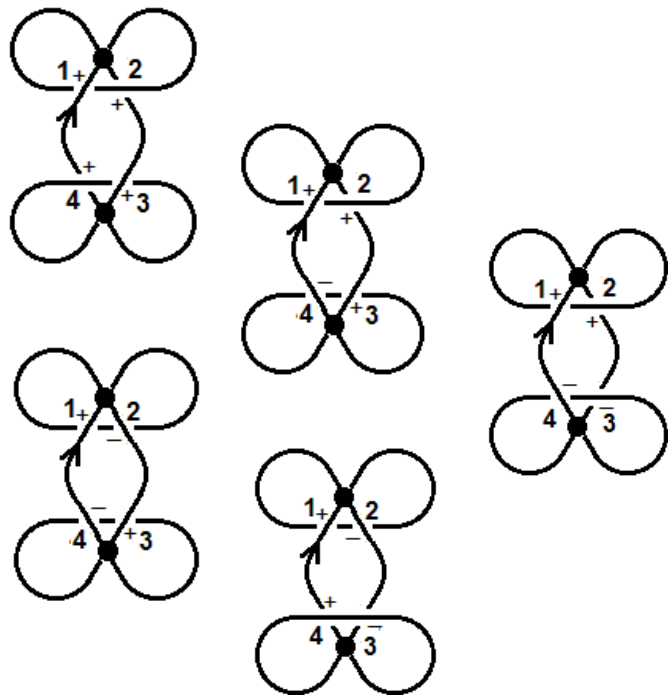
$O_1+O_2+U_1+U_2+U_3-U_4-O_3-O_4-$

**4Aa4**, Green's 4.77

$O_1+U_2-U_1+O_2-O_3+U_4-U_3+O_4-$

**4Aa5**, Green's 4.55

$O_1+U_2-U_1+O_2-U_3-O_4+O_3-U_4+$



Type b: 4 knots

Signatures  $+++-$   $+++-$   $++-+$  and  $+-$

**4Ab1**, Green's 4.3

$O_1+O_2+U_1+U_2+O_3+U_4+U_3+O_4+$

**4Ab2**, Green's 4.6

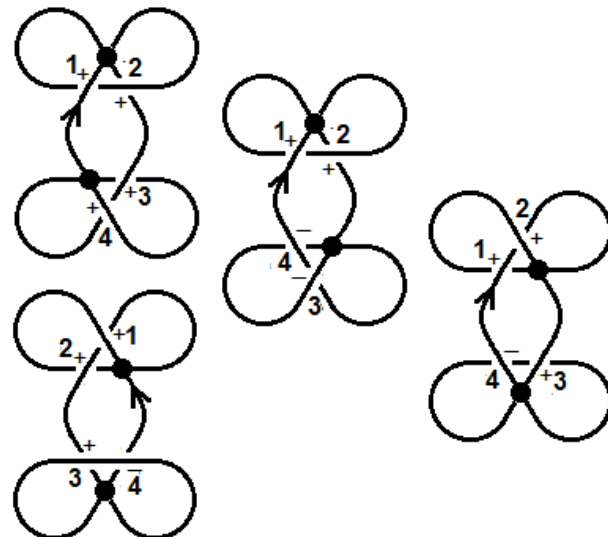
$O_1+O_2+U_1+U_2+O_3-U_4-U_3-O_4-$

**4Ab3**, Green's 4.74

$O_1+U_2+U_1+O_2+O_3+U_4-U_3+O_4-$

**4Ab4**, Green's 4.54

$O_1+U_2+U_1+O_2+U_3+O_4-O_3+U_4-$

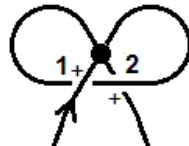


Type c: 5 knots

Signature  $++--$

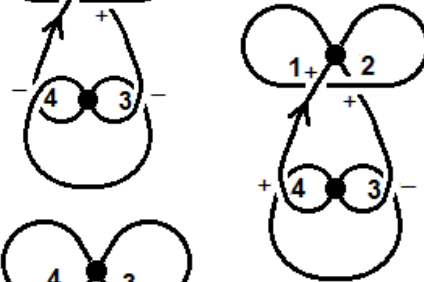
**4Ac1**, Green's 4.2

$O_1+O_2+U_1+U_2+O_3-O_4-U_3-U_4-$



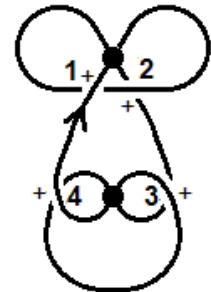
**4Ac2**, Green's 4.5

$O_1+O_2+U_1+U_2+O_3-U_4+U_3-O_4+$



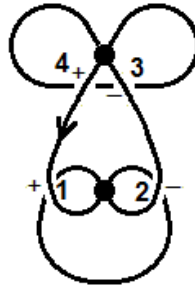
**4Ac3**, Green's 4.7

$O_1+O_2+U_1+U_2+U_3+U_4+O_3+O_4+$



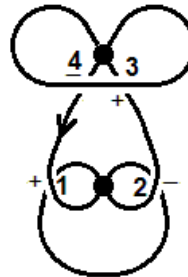
**4Ac4**, Green's 4.76

$O_1+U_2-U_1+O_2-O_3-U_4+U_3-O_4+$



**4Ac5**, Green's 4.56

$O_1+U_2-U_1+O_2-U_3+O_4-O_3+U_4-$

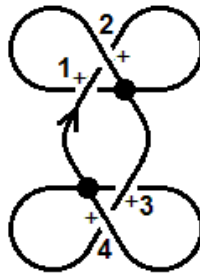


Type d: 2 knots

Signature  $+-+-$

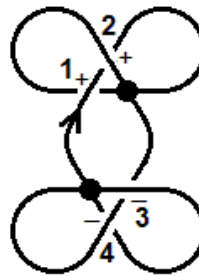
**4Ad1**, Green's 4.73

$O_1+U_2+U_1+O_2+O_3+U_4+U_3+O_4+$



**4Ad2**, not in Green's table

$O_1+U_2+U_1+O_2+U_3-O_4-O_3-U_4-$

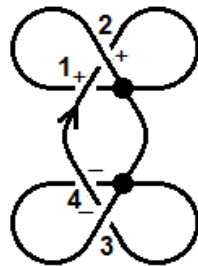


Type e: 2 knots

Signature  $+-++$

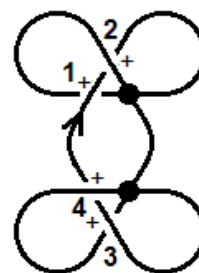
**4Ae1**, Green's 4.75

$O_1+U_2+U_1+O_2+O_3-U_4-U_3-O_4-$



**4Ae2**, Green's 4.53

$O_1+U_2+U_1+O_2+U_3+O_4+O_3+U_4+$



Class B: 6 types, 38 knots  
Sequence **12132434**

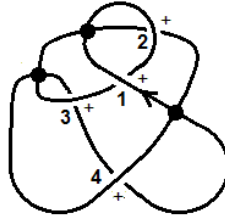


Type a: 6 knots

Signature ++++

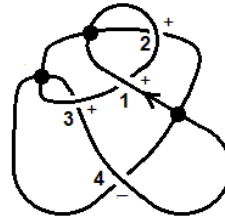
**4Ba1**, Green's 4.9

$O1+O2+U1+O3+U2+O4+U3+U4+$



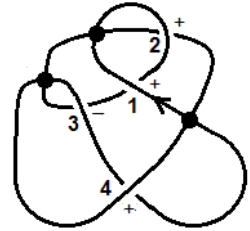
**4Ba2**, Green's 4.16

$O1+O2+U1+O3+U2+U4-U3+O4-$



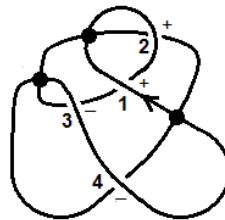
**4Ba3**, Green's 4.33

$O1+O2+U1+U3-U2+O4+O3-U4+$



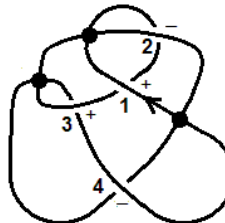
**4Ba4**, Green's 4.58

$O1+O2+U1+U3-U2+U4-O3-O4-$



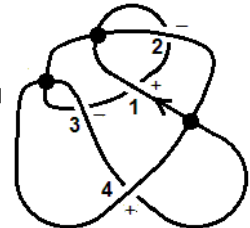
**4Ba5**, Green's 4.72

$O1+U2-U1+O3+O2-U4-U3+O4-$



**4Ba6**, Green's 4.52

$O1+U2-U1+U3-O2-O4+O3-U4+$

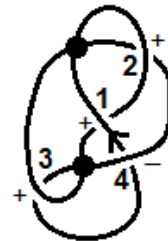


Type b: 8 knots

Signature +++- and +---

**4Bb1**, Green's 4.10

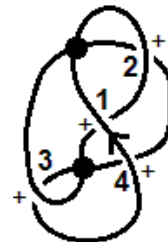
$O1+O2+U1+O3+U2+O4-U3+U4-$



**4Bb2**, Green's 4.15

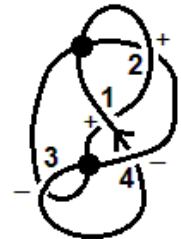
$O1+O2+U1+O3+U2+U4+U3+O4+$

This knot can be transformed by an R-move to **4Ca1**



**4Bb3**, Green's 4.32

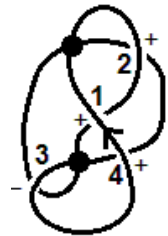
$O1+O2+U1+U3-U2+O4-O3-U4-$



**4Bb4**, Green's 4.38

$O1+O2+U1+U3-U2+U4+O3-O4+$

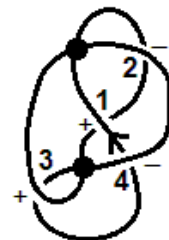
This knot can be transformed by an R-move to **4Ca2**



**4Bb5**, Green's 4.35

$O1+U2-U1+O3+O2-O4-U3+U4-$

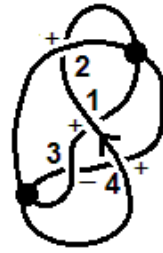
This knot can be transformed by an R-move to **4Ca7**



**4Bb6**, Green's 4.63

$$O1+U2+U1+O3-O2+U4+U3-O4+$$

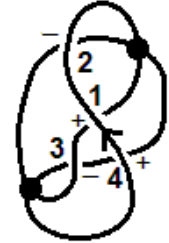
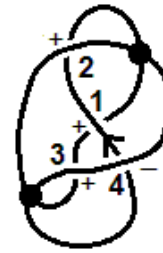
This knot can be transformed by an R-move to **4Ca3**



**4Bb7**, Green's 4.50

$$O1+U2+U1+U3+O2+O4-O3+U4-$$

This knot can be transformed by an R-move to **4Ca8**



**4Bb8**, Green's 4.20

$$O1+O2-U1+O3-U2-U4+U3-O4+$$

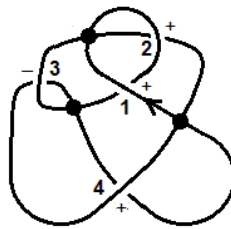
This knot can be transformed by an R-move to **4Ca4**

Type c: 8 knots

Signatures ++-+ and +-++

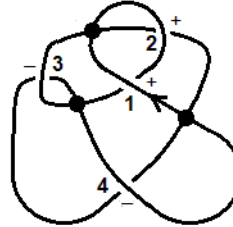
**4Bc1**, Green's 4.11

$$O1+O2+U1+O3-U2+O4+U3-U4+$$



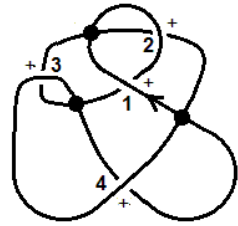
**4Bc2**, Green's 4.17

$$O1+O2+U1+O3-U2+U4-U3-O4-$$



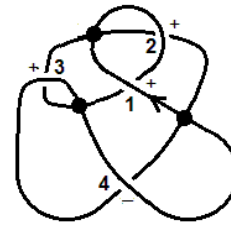
**4Bc3**, Green's 4.29

$$O1+O2+U1+U3+U2+O4+O3+U4+$$



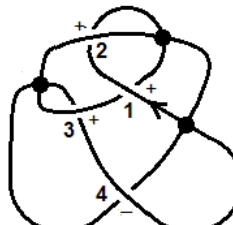
**4Bc4**, Green's 4.57

$$O1+O2+U1+U3+U2+U4-O3+O4-$$



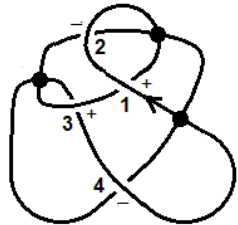
**4Bc5**, Green's 4.70

$$O1+U2+U1+O3+O2+U4-U3+O4-$$



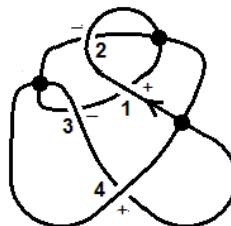
**4Bc6**, Green's 4.19

$$O1+O2-U1+O3+U2-U4-U3+O4-$$



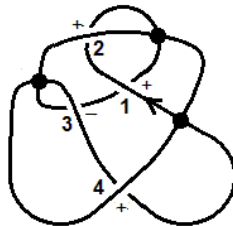
**4Bc7**, Green's 4.34

$$O1+O2-U1+U3-U2-O4+O3-U4+$$



**4Bc8**, Green's 4.49

$$O1+U2+U1+U3-O2+O4+O3-U4+$$



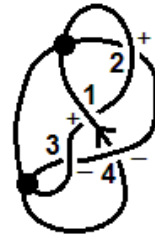


Type d: 4 knots

Signature  $+++--$

**4Bd1**, Green's 4.12

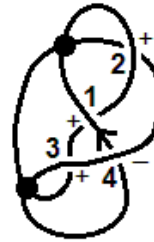
$$O_1+O_2+U_1+O_3-U_2+O_4-U_3-U_4-$$



**4Bd2**, Green's 4.36

$$O_1+O_2+U_1+U_3+U_2+O_4-O_3+U_4-$$

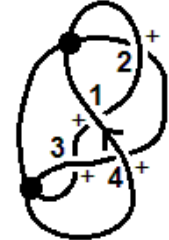
This knot can be transformed by an R-move to **4Cb4**



**4Bd3**, Green's 4.37

$$O_1+O_2+U_1+U_3+U_2+U_4+O_3+O_4+$$

This knot can be transformed by an R-move to **4Cb1**



**4Bd4**, Green's 4.64

$$O_1+U_2-U_1+O_3-O_2-U_4+U_3-O_4+$$

This knot can be transformed by an R-move to **4Cb2**

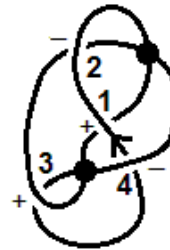


Type e: 6 knots

Signature  $+--+$

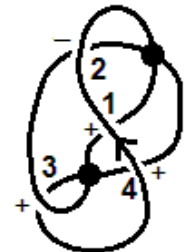
**4Be1**, Green's 4.13

$$O_1+O_2-U_1+O_3+U_2-O_4-U_3+U_4-$$



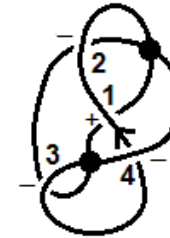
**4Be2**, Green's 4.18

$$O_1+O_2-U_1+O_3+U_2-U_4+U_3+O_4+$$



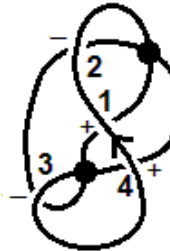
**4Be3**, Green's 4.31

$$O_1+O_2-U_1+U_3-U_2-O_4-O_3-U_4-$$



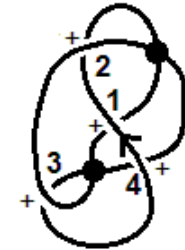
**4Be4**, Green's 4.60

$$O_1+O_2-U_1+U_3-U_2-O_4-O_3-U_4-$$



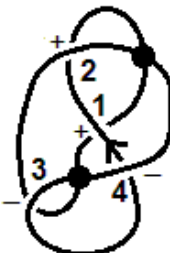
**4Be5**, Green's 4.69

$$O_1+U_2+U_1+O_3+O_2+U_4+U_3+O_4+$$



**4Be6**, Green's 4.51

$$O_1+U_2+U_1+U_3-O_2+O_4-O_3-U_4-$$



Type f: 6 knots

Signature  $+-+-$

**4Bf1**, Green's 4.14

$O1+O2-U1+O3-U2-O4+U3-U4+$

**4Bf2**, Green's 4.21

$O1+O2-U1+O3-U2-U4-U3-O4-$

**4Bf3**, Green's 4.30

$O1+O2-U1+U3+U2-O4+O3+U4+$

**4Bf4**, Green's 4.59

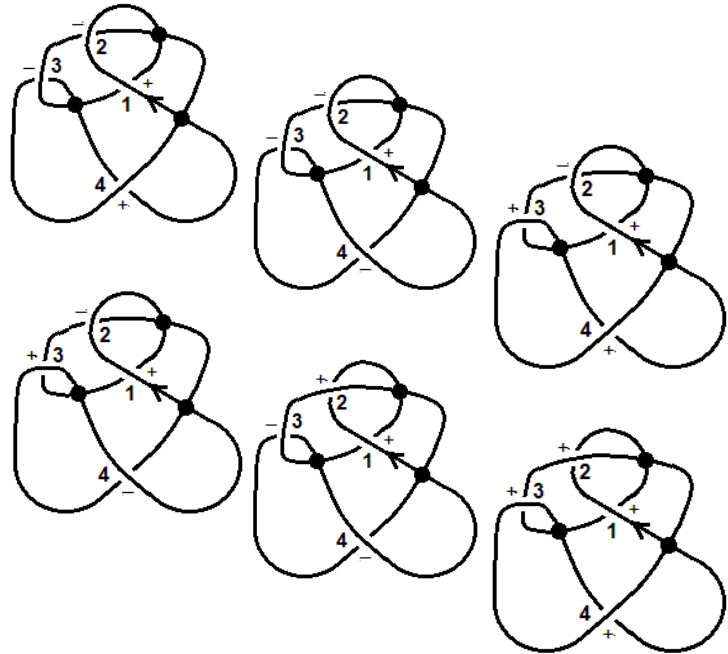
$O1+O2-U1+U3+U2-U4-O3+O4-$

**4Bf5**, Green's 4.71

$O1+U2+U1+O3-O2+U4-U3-O4-$

**4Bf6**, Green's 4.48

$O1+U2+U1+U3+O2+O4+O3+U4+$



Class C: 4 types, 24 knots

Sequence **12134234**



Type a: 8 knots

Signatures  $++++$  and  $+-+-$

**4Ca1**, Green's 4.15

$O1+O2+U1+O3+O4+U2+U3+U4+$

This knot can be transformed by an R-move to **4Bb2**

**4Ca2**, Green's 4.38

$O1+O2+U1+O3+U4-U2+U3+O4-$

This knot can be transformed by an R-move to **4Bb4**

**4Ca3**, Green's 4.63

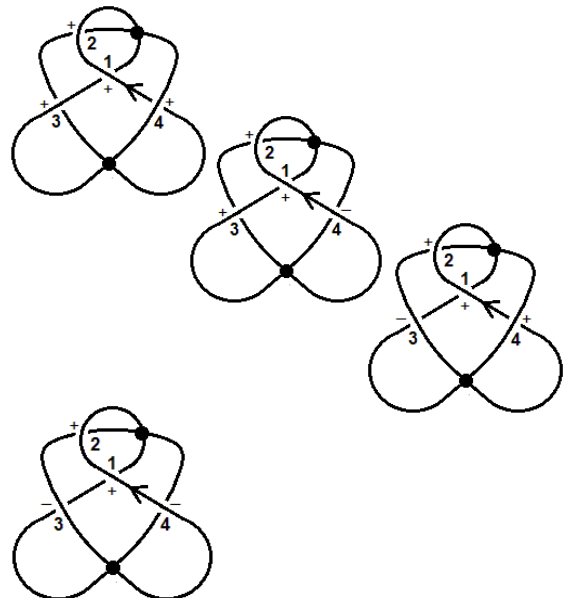
$O1+O2+U1+U3-O4+U2+O3-U4+$

This knot can be transformed by an R-move to **4Bb6**

**4Ca4**, Green's 4.20

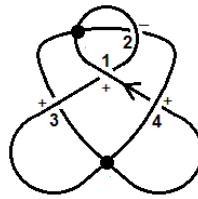
$O1+O2+U1+U3-U4-U2+O3-O4-$

This knot can be transformed by an R-move to **4Bb8**



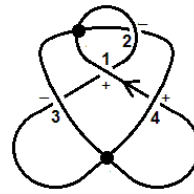
**4Ca5**, Green's 4.23

$$01+02-U1+03+04+U2-U3+U4+$$



**4Ca6**, Green's 4.67

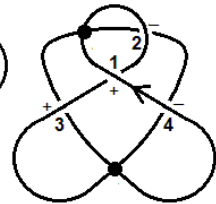
$$01+02-U1+U3-04+U2-03-U4+$$



**4Ca7**, Green's 4.35

$$01+02-U1+03+U4-U2-U3+04-$$

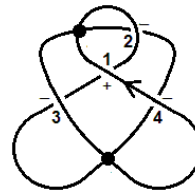
This knot can be transformed by an R-move to **4Bb5**



**4Ca8**, Green's 4.50

$$01+02-U1+U3-U4-U2-03-04-$$

This knot can be transformed by an R-move to **4Bb7**



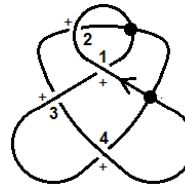
Type b: 4 knots

Signatures  $+++ -$  and  $+++ +$

**4Cb1**, Green's 4.37

$$01+02+U1+03+U4+U2+U3+04+$$

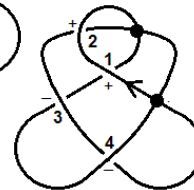
This knot can be transformed by an R-move to **4Bd3**



**4Cb2**, Green's 4.64

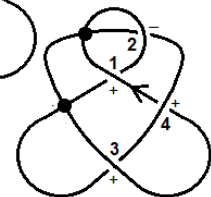
$$01+02+U1+U3-04-U2+03-U4-$$

This knot can be transformed by an R-move to **4Bd4**



**4Cb3**, Green's 4.65

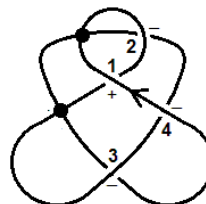
$$01+02-U1+U3+04+U2-03+U4+$$



**4Cb4**, Green's 4.36

$$01+02-U1+03-U4-U2-U3-04-$$

This knot can be transformed by an R-move to **4Bd2**

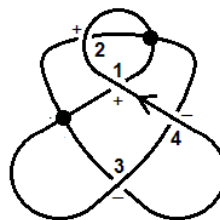


Type c: 4 knots

Signatures  $++-+$  and  $++- -$

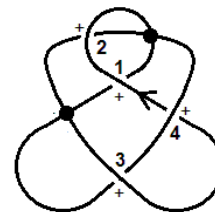
**4Cc1**, Green's 4.40

$$01+02+U1+03-U4-U2+U3-04-$$



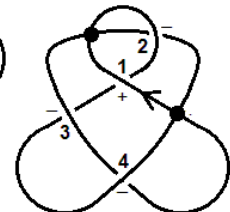
**4Cc2**, Green's 4.61

$$01+02+U1+U3+04+U2+03+U4+$$



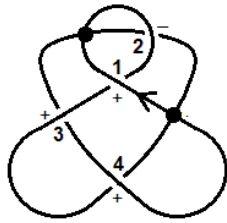
**4Cc3**, Green's 4.68

$$01+02-U1+U3-04-U2-03-U4-$$



**4Cc4**, Green's 4.41

$O1+O2-U1+O3+U4+U2-U3+O4+$

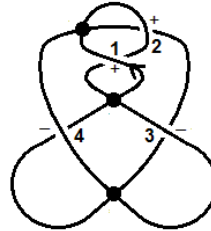


Type d: 8 knots

Signatures  $++--$  and  $++--$

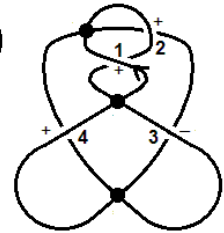
**4Cd1**, Green's 4.22

$O1+O2+U1+O3-O4-U2+U3-U4-$



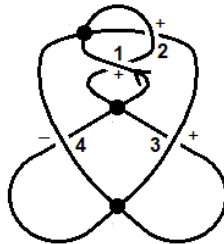
**4Cd2**, Green's 4.39

$O1+O2+U1+O3-U4+U2+U3-O4+$



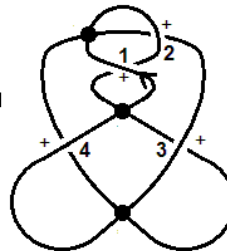
**4Cd3**, Green's 4.62

$O1+O2+U1+U3+O4-U2+O3+U4-$



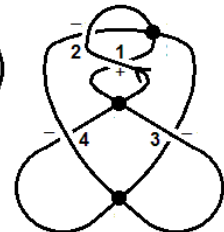
**4Cd4**, Green's 4.78

$O1+O2+U1+U3+U4+U2+O3+O4+$



**4Cd5**, Green's 4.24

$O1+O2-U1+O3-O4-U2-U3-U4-$



**4Cd6**, Green's 4.66

$O1+O2-U1+U3+O4-U2-O3+U4-$



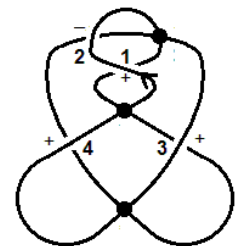
**4Cd7**, Green's 4.42

$O1+O2-U1+O3-U4+U2-U3-O4+$



**4Cd8**, Green's 4.79

$O1+O2-U1+U3+U4+U2-O3+O4+$



Class D: 3 types, 11 knots

Sequence **12134243**

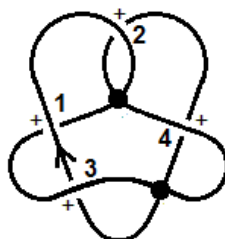


Type a: 4 knots

Signatures  $++++$   $++-+$   $-+++$  and  $+-++$

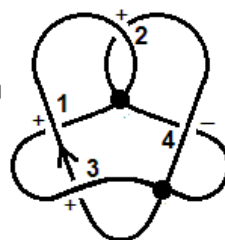
**4Da1**, Green's 4.25

$O1+O2+U1+O3+O4+U2+U4+U3+$



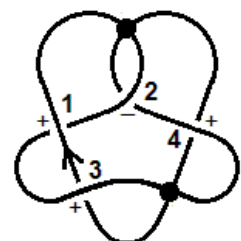
**4Da2**, Green's 4.44

$O1+O2+U1+O3+U4-U2+O4-U3+$



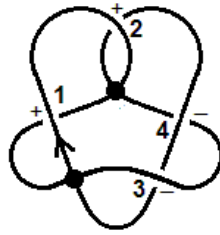
**4Da3**, Green's 4.27

$O1+O2-U1+O3+O4+U2-U4+U3+$



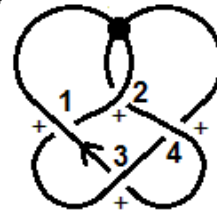
**4Da4**, Green's 4.46

$O1+O2+U1+O3-U4-U2+O4-U3-$



Type b: 1 knot

Signatures  $+++-$  and  $+-+-$



**4Db1**, Green's 4.43

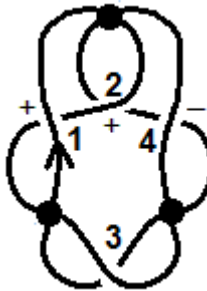
$O1+O2+U1+O3+U4+U2+O4+U3+$

Type c: 6 knots

Signatures  $+++-$  and  $+-+-$

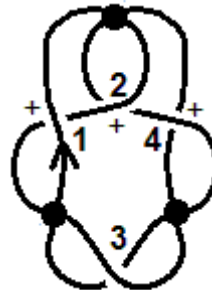
**4Dc1**, Green's 4.26

$O1+O2+U1+O3-O4-U2+U4-U3-$



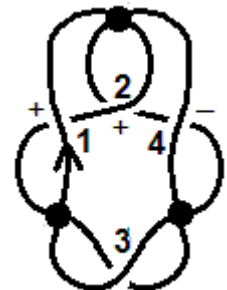
**4Dc2**, Green's 4.45

$O1+O2+U1+O3-U4+U2+O4+U3-$



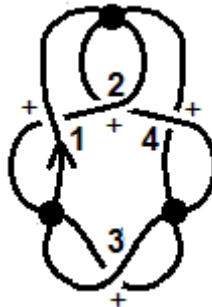
**4Dc3**, Green's 4.28

$O1+O2+U1+U3+O4-U2+U4-O3+$



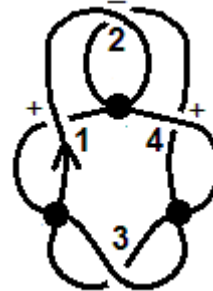
**4Dc4**, Green's 4.80

$O1+O2+U1+U3+U4+U2+O4+O3+$



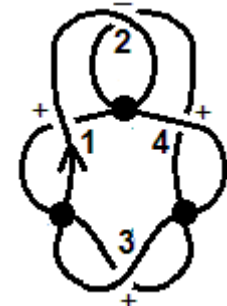
**4Dc5**, Green's 4.47

$O1+O2-U1+O3-U4+U2-O4+U3-$



**4Dc6**, Green's 4.81

$O1+O2-U1+U3+U4+U2-O4+O3+$



Class E: 5 types, 14 knots

Sequence **12314234**

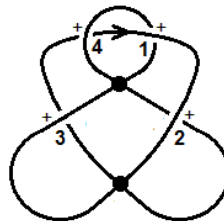


Type a: 3 knots

Signatures  $++++$  and  $----$

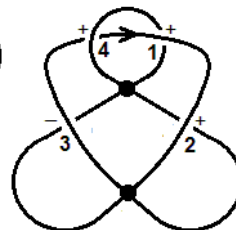
**4Ea1**, Green's 4.82

$O1+O2+O3+U1+O4+U2+U3+U4+$



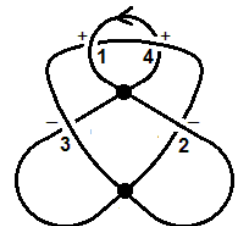
**4Ea2**, Green's 4.96

$O1+O2+U3-U1+O4+U2+O3-U4+$



**4Ea3**, Green's 4.84

$O1+O2-O3-U1+O4+U2-U3-U4+$



Type b: 6 knots

Signatures +++- and +---

**4Eb1**, Green's 4.83

$$O1+O2+O3+U1+O4-U2+U3+U4-$$

**4Eb2**, Green's 4.87

$$O1+O2+O3+U1+U4+U2+U3+O4+$$

**4Eb3**, Green's 4.97

$$O1+O2+U3-U1+O4-U2+O3-U4-$$

**4Eb4**, Green's 4.93

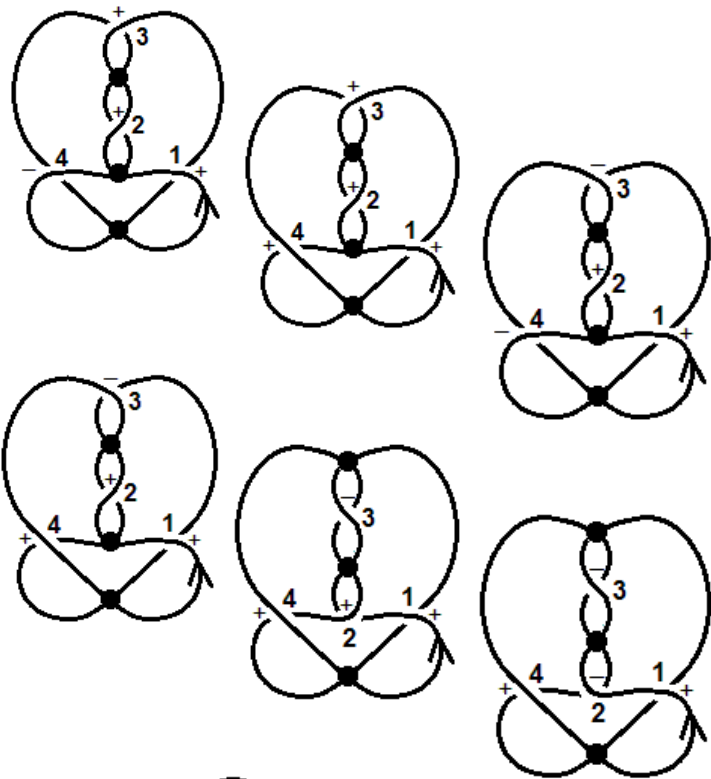
$$O1+O2+U3-U1+U4+U2+O3-O4+$$

**4Eb5**, Green's 4.103

$$O1+U2+O3-U1+U4+O2+U3-O4+$$

**4Eb6**, Green's 4.88

$$O1+O2-O3-U1+U4+U2-U3-O4+$$



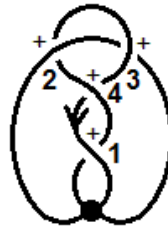
Type c: 1 knot

Signatures +++- and +---

**4Ec1**, Green's 4.94

$$O1+O2+U3+U1+O4+U2+O3+U4+$$

This knot can be transformed by R-moves to **4Ee1**



Type d: 3 knots

Signature +++-

**4Ed1**, Green's 4.95

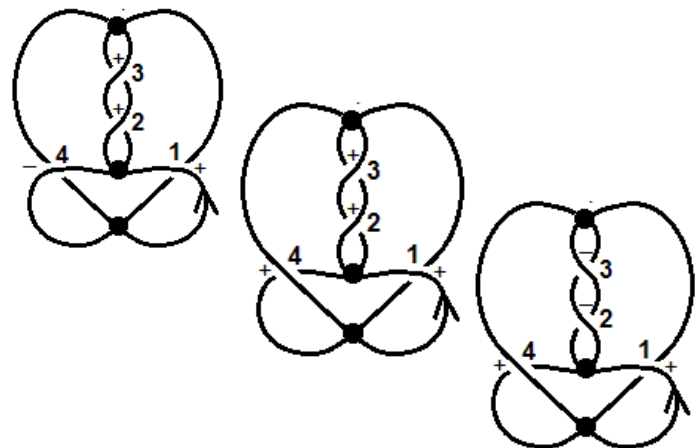
$$O1+O2+U3+U1+O4-U2+O3+U4-$$

**4Ed2**, Green's 4.92

$$O1+O2+U3+U1+U4+U2+O3+O4+$$

**4Ed3**, Green's 4.104

$$O1+U2-O3-U1+U4+O2-U3-O4+$$



Type e: 1 knot

Signature +++-

**4Ee1**, Green's 4.94

$$O1+U2+O3+U1+U4+O2+U3+O4+$$

This knot can be transformed by R-moves to **4Ec1**



Class F: 4 types, 10 knots  
Sequence **12314324**



Type a: 2 knots

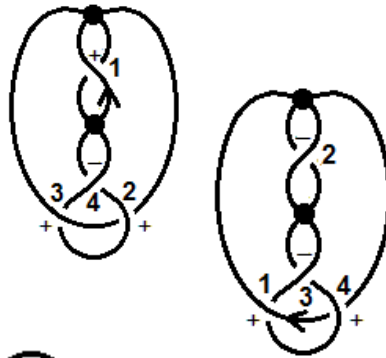
Signatures  $++++$   $++--$   $-+-+$  and  $----$

**4Fa1**, Green's 4.106

$O1+U2+O3+U1+O4-U3+O2+U4-$

**4Fa2**, Green's 4.86

$O1+O2-O3-U1+O4+U3-U2-U4+$



Type b: 5 knots

Signatures  $+++-$  and  $----$

**4Fb1**, Green's 4.85

$O1+O2+O3+U1+O4-U3+U2+U4-$

**4Fb2**, Green's 4.89

$O1+O2+O3+U1+U4+U3+U2+O4+$

**4Fb3**, Green's 4.98

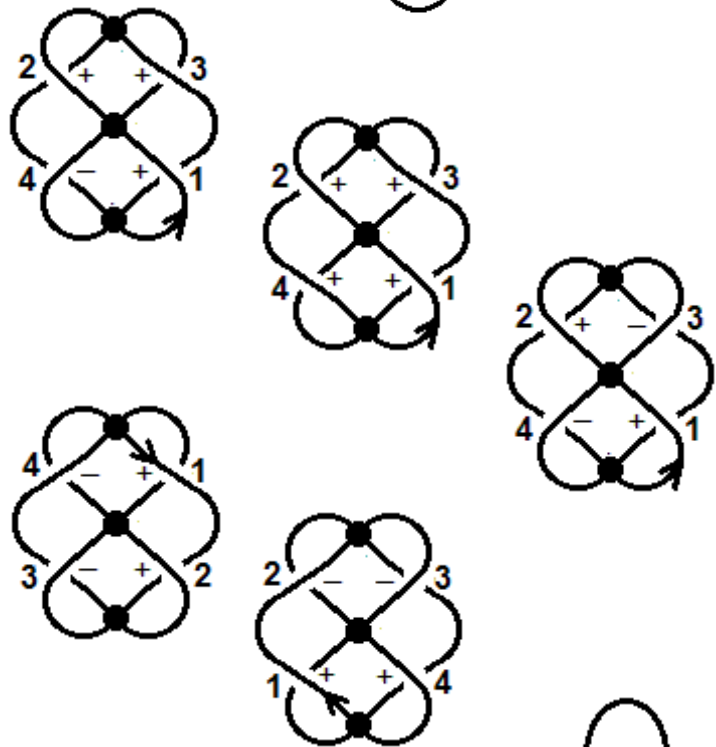
$O1+O2+U3-U1+O4-O3-U2+U4-$

**4Fb4**, Green's 4.107

$O1+U2+O3-U1+O4-U3-O2+U4-$

**4Fb5**, Green's 4.90

$O1+O2-O3-U1+U4+U3-U2-O4+$

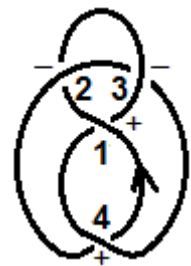


Type c: 1 knot

Signature  $+-+$

**4Fc1**, Green's 4.108

$O1+U2-O3-U1+O4+U3-O2-U4+$



Type d: 2 knots

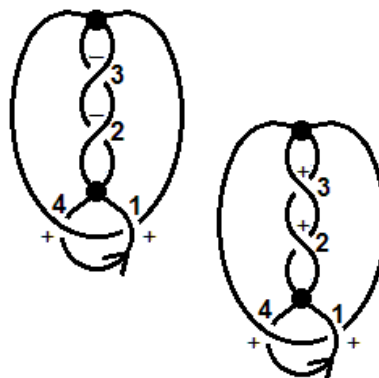
Signature  $+++$

**4Fd1**, Green's 4.99

$O1+O2-U3-U1+O4+O3-U2-U4+$

**4Fd2**, Green's 4.105

$O1+U2+O3+U1+O4+U3+O2+U4+$



Class G: 2 types, 4 knots  
 Sequence **12341234**



Type a: 3 knots

Signatures  $++++$   $+++-$   $++--$  and  $+---$

**4Ga1**, Green's 4.91

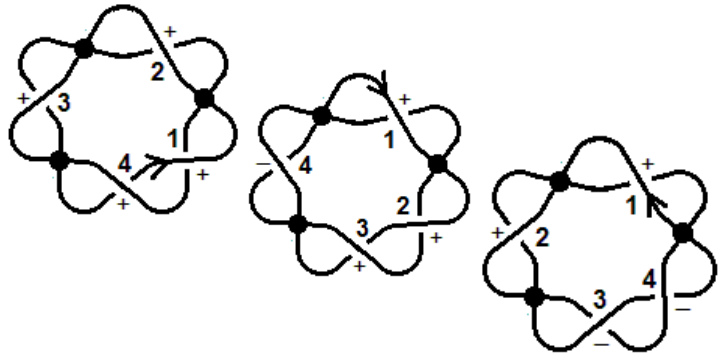
$O1+O2+O3+O4+U1+U2+U3+U4+$

**4Ga2**, Green's 4.101

$O1+O2+U3+O4-U1+U2+O3+U4-$

**4Ga3**, Green's 4.102

$O1+O2+U3-O4-U1+U2+O3-U4-$

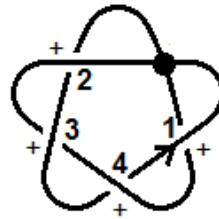


Type b: 1 knot

Signatures  $+-+-$   $++++$   $+-+-$  and  $+-+-$

**4Gb1**, Green's 4.100

$O1+O2+U3+O4+U1+U2+O3+U4+$





## 8. Summary

Green's numbers	This note
2.1	2Aa1
3.1	3Ab1
3.2	3Aa1
3.3	3Ab2
3.4	3Ab3
3.5	3Ba2
3.6	3Bb1
3.7	3Ba1
4.1	4Aa1
4.2	4Ac1
4.3	4Ab1
4.4	4Aa2
4.5	4Ac2
4.6	4Ab2
4.7	4Ac3
4.8	4Aa3
4.9	4Ba1
4.10	4Bb1
4.11	4Bc1
4.12	4Bd1
4.13	4Be1
4.14	4Bf1
4.15	4Bb2/4Ca1
4.16	4Ba2
4.17	4Bc2
4.18	4Be2
4.19	4Bc6
4.20	4Bb8/4Ca4
4.21	4Bf2
4.22	4Cd1
4.23	4Ca5
4.24	4Cd5
4.25	4Da1
4.26	4Dc1
4.27	4Da3
4.28	4Dc3
4.29	4Bc3
4.30	4Bf3
4.31	4Be3
4.32	4Bb3
4.33	4Ba3

Green's numbers	This note
4.34	4Bc7
4.35	4Bb5/4Ca7
4.36	4Bd2/4Cb4
4.37	4Bd3/4Cb1
4.38	4Bb4/4Ca2
4.39	4Cd2
4.40	4Cc1
4.41	4Cc4
4.42	4Cd7
4.43	4Db1
4.44	4Da2
4.45	4Dc2
4.46	4Da4
4.47	4Dc5
4.48	4Bf6
4.49	4Bc8
4.50	4Bb7/4Ca8
4.51	4Be6
4.52	4Ba6
4.53	4Ae2
4.54	4Ab4
4.55	4Aa5
4.56	4Ac5
4.57	4Bc4
4.58	4Ba4
4.59	4Bf4
4.60	4Be4
4.61	4Cc2
4.62	4Cd3
4.63	4Bb6/4Ca3
4.64	4Bd4/4Cb2
4.65	4Cb3
4.66	4Cd6
4.67	4Ca6
4.68	4Cc3
4.69	4Be5
4.70	4Bc5
4.71	4Bf5
4.72	4Ba5
4.73	4Ad1
4.74	4Ab3
4.75	4Ae1
4.76	4Ac4

Green's numbers	This note
4.77	4Aa4
4.78	4 Cd4
4.79	4Cd8
4.80	4Dc4
4.81	4Dc6
4.82	4Ea1
4.83	4Eb1
4.84	4Ea3
4.85	4Fb1
4.86	4Fa2
4.87	4Eb2
4.88	4Eb6
4.89	4Fb2
4.90	4Fb5
4.91	4Ga1
4.92	4Ed2
4.93	4Eb4
4.94	4Ec1/4Ee1
4.95	4Ed1
4.96	4Ea2
4.97	4Eb3
4.98	4Fb3
4.99	4Fd1
4.100	4Gb1
4.101	4Ga2
4.102	4Ga3
4.103	4Eb5
4.104	4Ed3
4.105	4Fd2
4.106	4Fa1
4.107	4Fb4
4.108	4Fc1
4.00	4Ad2

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